Schienbein et al.
Migration of cells

Cells move randomly while also guided in direction of inflammation, chemical signal, or electric field. Movement observed in wound healing, embryogenesis, and granulocytes. Can also modify this for chemotaxis.
Langevin equation can describe this

\[ \dot{v} + \gamma_v v = \gamma_v v_s + \Gamma(t) \]

White noise defined by the following properties

\[ \langle \Gamma(t) \rangle = 0 \]

\[ \langle \Gamma(t) \Gamma(t') \rangle = q_v \, \delta(t - t') \]
Langevin movement corresponds to Kolmogorov forward equation/Fokker-Planck for probability

$$\frac{\partial W(v, t)}{\partial t} = \left( \gamma_v \frac{\partial}{\partial v} (v - v_s) + \frac{1}{2} q_v \frac{\partial^2}{\partial v^2} \right) W(v, t)$$

Steady state solution

$$W_{st}(v) = W_0 \exp\left( -\frac{\gamma_v}{q_v} (v - v_s)^2 \right)$$
Solution to Langevin and correlation function

\[ v(t) = v_0 e^{-\gamma_v t} + \int_0^t \left[ \gamma_v v_s + \Gamma(t') \right] e^{-\gamma_v (t-t')} \, dt' \]

Correlation in limit of long times for \( t_1 \) and \( t_2 \)

\[ \langle v(t_1)v(t_2) \rangle = v_s^2 + \frac{q_v}{2\gamma_v} e^{-\gamma_v|t_2-t_1|}. \]

Relative Correlation

\[ g_v(\infty) = \frac{v_s^2}{v_s^2 + \frac{q_v}{2\gamma_v}}. \]
Similar equations and analyses apply for the distribution of angles

Langevin

\[ \dot{\phi}(t) = -k_p E \sin \phi(t) + \Gamma(t). \]

Steady state probability distribution

\[ W_{st}(\phi) = \frac{\exp^{K_G E \cos \phi}}{\int_0^{2\pi} \exp^{K_G E \cos \phi} \, d\phi}. \]
Distribution for different electric fields
Expected value for cosine angle: theory and data
Expected square of displacement

\[ \langle x(t)^2 \rangle = \left( \frac{A}{\Lambda} + \frac{B}{\lambda} \right) t - \frac{A}{\Lambda^2} (1 - e^{-\Lambda t}) - \frac{B}{\lambda^2} (1 - e^{-\lambda t}); \]

\[ \lambda = \lambda_1 = \frac{1}{\tau_\phi} \]

\[ \Lambda = \gamma_v + \lambda_1 = \frac{1}{\tau_v} + \frac{1}{\tau_\phi} \]

\[ A = \frac{q_v}{2 \gamma_v} \]

\[ B = v_s^2. \]
Predictions and Data

\[ \frac{1}{2} \left( \langle x^2 \rangle + \langle y^2 \rangle \right) \]
Time evolution of probability distribution with no electric field
Time evolution of probability distribution with electric field turned on
Some discrepancies between theory and data for full case
Sources of discrepancy

1. Not at steady state during shift in electric field
2. Noise may not be white noise
3. Deterministic part may have frequency dependence
4. There is some weak memory to the “noise”